MA 262 EXAM II REVIEW PROBLEMS

SOLUTIONS

References to page numbers of the review problems are not quite right

1, Page 1) To find the dimension of the space spanned by a set of given vectors we write them as columns or rows of a matrix, reduce it to an echelon form and count the number of non-zero rows. In this case we write

$$\begin{bmatrix} 1 & 3 & 5 & -2 \\ -1 & 1 & -3 & 4 \\ 1 & 7 & 9 & 2 \\ -1 & 1 & 1 & 8 \end{bmatrix} \xrightarrow[R_1+R_4]{R_1+R_2,\ -R_1+R_3} \begin{bmatrix} 1 & 3 & 5 & -2 \\ 0 & 4 & 2 & 2 \\ 0 & 4 & 4 & 4 \\ 0 & 4 & 6 & 6 \end{bmatrix} \xrightarrow[-R_2+R_4]{-R_2+R_3}$$

$$\begin{bmatrix} 1 & 3 & 5 & -2 \\ 0 & 4 & 2 & 2 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 4 & 4 \end{bmatrix} \xrightarrow[R_2/2,\ R_3/2]{-2R_3+R_4} \begin{bmatrix} 1 & 3 & 5 & -2 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The matrix has 3 non-zero rows so the space spanned by the four vectors has dimension 3.

C

Page 1 (2)

2) The vectors form a basis if they are Linearly

Page 1 Independent, or if the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 2 & 3 \\ 0 & 2 & k \end{bmatrix}$$ has an inverse.

There are several ways of doing this, but one of them is to compute its determinant

$$\det A = \det \begin{bmatrix} 1 & 1 & 0 \\ -1 & 2 & 3 \\ 0 & 2 & k \end{bmatrix} = \det \begin{bmatrix} 1 & 1 & 0 \\ 0 & 3 & 3 \\ 0 & 2 & k \end{bmatrix}$$

$= 3k - 6$. If $\det A \neq 0$ A has an inverse. and the vectors are L.I.

So $3k - 6 \neq 0$; $\boxed{k \neq 2}$ (D)

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3) $C = A_1 \cdot B_1$ So $\det C = \det A_1 \cdot \det B_1$.

Page 2 $\det A_1 = 2 \det A$, $\det B_1 = 3 \det B$

$\det B_1 = 18$.

$\det A_1 = 10$

$\boxed{\det C = 180}$ (A)

4) Page 2 $\quad y = e^{rx} \qquad r^2 - 2r + 1 = (r-1)^2 = 0$ (3)

So the general solution of the equation

is $\quad y(x) = A e^{x} + B x e^{x}. \qquad y(0) = A = 1$

$y'(x) = A e^{x} + B e^{x} + B x e^{x} \qquad y'(0) = A + B = -1$

So $\quad B = -2 \quad$ and $\quad y(x) = e^{x} - 2x e^{x}.$

$$\boxed{y(1/2) = e^{1/2} - e^{1/2} = 0}$$

(A)

5 Page 3) $\quad$ Let $\quad y(x) = x^{r}; \qquad y' = r x^{r-1}$

$y'' = r(r-1) x^{r-2}$

So: $\quad x^2 y'' + 4x y' + 2y = r(r-1) x^{r} + 4r x^{r} + 2x^{r}$

$= x^{r}\left(r(r-1) + 4r + 2 \right) = 0 \qquad$ So $\quad r^2 - r + 4r + 2 = 0$

$r^2 + 3r + 2 = (r+1)(r+2) = 0$

So we have two solution: $\quad y_1(x) = x^{-1}, \; y_2(x) = x^{-2}$

$$W(y_1, y_2) = \det \begin{bmatrix} x^{-1} & x^{-2} \\ -x^{-2} & -2x^{-3} \end{bmatrix} = -2x^{-4} + x^{-4} = -x^{-4} \neq 0$$

for $\quad x > 0$

$$\boxed{y(x) = C_1 x^{-1} + C_2 x^{-2}}$$

(A)

6 Page 3) As above, try to find solutions of the form $y = x^r$: (4

$$r(r-1)x^r - 2rx^r + 2x^r = 0$$

$$x^r\left(r(r-1) - 2r + 2 \right) = x^r\left(r^2 - 3r + 2 \right) = 0$$

$$(r-1)(r-2) = 0 \qquad y_1 = x \; ; \; y_2 = x^2.$$

roots are $r=1, r=2$

$y(x) = C_1 x + C_2 x^2$. $\qquad y(1) = C_1 + C_2 = 2 \qquad C_1 = C_2 = 1$

$y'(x) = C_1 + 2C_2 x \qquad y'(1) = C_1 + 2C_2 = 3$

$y(x) = x + x^2$. $\qquad y(2) = 2 + 4 = 6$. C

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7) The roots of the characteristic polynomial

Page 4 are $r_1 = 1+2i$, $r_2 = 1-2i$, $r_3 = 4$

So we have 3 L.I solutions of the equation: $y_1(x) = e^x \cos 2x$; $y_2 = e^x \sin 2x$

$y_3 = e^{4x}$.

General Solution

$$\boxed{y(x) = C_1 e^x \cos 2x + C_2 e^x \sin 2x + C_3 e^{4x}}$$

A

8 Page 4) roots are: $r=1$ multiplicity 2

$r=-1$ multiplicity 2.

$r=2i$, $r=-2i$ also with multiplicity 2.

L.I Solution: $y_1 = e^x$, $y_2 = x e^x$

$y_3 = e^{-x}$, $y_4 = x e^{-x}$

$y_5 = \cos 2x$, $y_6 = \sin 2x$

$y_7 = x \cos 2x$, $y_8 = x \sin 2x$.

The general solution is a Linear combination of these:

A

9 Page 5) $W(x, \sin x, \cos x) = \det \begin{bmatrix} x & \sin x & \cos x \\ 1 & \cos x & -\sin x \\ 0 & -\sin x & -\cos x \end{bmatrix}$

$= x(-\cos^2 x - \sin^2 x) - 1(-\sin x \cos x + \sin x \cos x) = -x.$

B

6 Page 6) A system $AX = B$ has infinitely many solutions if A does not have an inverse. In this case the homogeneous system $AX = 0$ has infinitely many solutions. D

REMARK: We have not yet defined what a defective matrix is

7 Page 7) If $Ap = b$ and $Aq = b$

$$b = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \text{ then } A(2p) = 2Ap = 2b$$

$$A(2p) - Aq = 2b - b = b$$

$$A(2p - q) = b$$ So $2p - q$ is also a solution. C

#8 Page 8) Since A is a 3×3 matrix, it would not be so hard just to compute A^{-1} and find b_{32}.

However it is easier to do this using Cramer's Rule: $A^{-1} = \frac{1}{\det A} \cdot C^T$

where C is the matrix of cofactors of A.

So $b_{32} = \frac{1}{\det A} \cdot C_{23}$ (it is C_{23} because we are dealing with C^T).

$C_{23} = (-1)^{2+3} \cdot$ det of the matrix obtained from A by deleting the 2nd row and 3rd column.

$$A = \begin{array}{c} \text{Column} \\ \left[\begin{array}{ccc} 1 & 2 & 1 \\ -1 & 4 & 1 \\ 2 & -4 & 0 \end{array}\right] \end{array}$$

$$C_{23} = (-1)^{2+3} \det \begin{bmatrix} 1 & 2 \\ 2 & -4 \end{bmatrix} = 8.$$

$$\det A = \det \begin{bmatrix} 1 & 2 & 1 \\ 0 & 6 & 2 \\ 0 & -8 & -2 \end{bmatrix} = -12 + 16 = 4.$$

$$b_{32} = 8/4 = 2$$

D

10, Page 9 | A is a $m \times n$ matrix and the equation $Ax = b$ has infinitely many solution.

(1) $m \leq n$. Not so ~~as~~ we can have a system with 3 equation and two variables with infinitely many solution:

$$x_1 + x_2 = 1$$
$$2x_1 + 2x_2 = 2$$
$$3x_1 + 3x_2 = 3$$

(2) $n \geq m$. Not so either $x_1 + x_2 = 0$
$m = 1$, $n = 2$ has infinitely many solution

(c) Rank of $A = n$ Not so $A = \begin{bmatrix} 1 & 1 \end{bmatrix}$, rank of $A = 1$, $n = 2$. infinitely many solution

(3) If $Ax = b$ has infinitely many solution its reduced row echelon form has at least one row of zero. So the null set of A has dimension ≥ 1

$$\dim \text{Null } A = n - \text{Rank } A \geq 1$$

$$\boxed{\text{Rank } A < n}$$

B

(4) det A may not be defined

14, [redacted])

Page 10

$$A = \begin{bmatrix} 1 & -2 & 0 & 4 \\ 3 & 1 & 1 & 0 \\ -1 & -5 & -1 & 8 \\ 3 & 8 & 2 & -12 \end{bmatrix} \xrightarrow[\substack{R_1+R_3 \\ -3R_1+R_3}]{-3R_1+R_2}$$

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 7 & 1 & -12 \\ 0 & -7 & -8 & 12 \\ 0 & 14 & 2 & -24 \end{bmatrix} \xrightarrow[-2R_2+R_4]{R_2+R_3}$$

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 7 & 1 & -12 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Rank = # of non-zero rows of an echelon form of A. = $\underline{\underline{2}}$

C

#18, Page 12)

(A) B is in reduced row echelon form. So A is false.

(B) The determinant of A is a non-zero multiple of det B, but det B = 0, So det A = 0

(C) dim of Null space = # of columns − Rank

Rank = # of non zero rows. = 3

dim of Null space = 4 − 3 = 1

(D) dimension of Column space = Rank = 3.

(E) Since B has a row of zeros a system $A.x = b$ either has no solution or infinitely many solution. This is correct

D.

#11, Page 12) If A is a 5×6 matrix

dim Null A = # of Columns − Rank(A) = 4

$6 - \text{Rank}(A) = 4$ Rank(A) = 2.

Dimension of Column space = Dim of Raw space = Rank(A) = 2. E

#4, Page 13)

A) Is false. $\det(A+B) \neq \det(A) + \det(B)$

B) $\det A^T = \det A$, So B is false

c) $AB = 0$ implies $A = 0$ a $B = 0$ is also false. It is easy to see for 2×2 matrices $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

D) Also false., $\det A = 0$ means that the raws are linearly dependent but not necessarily proportional

→ E $\det(-A) = (-1)^5 \det A = -\det A$

#7, Page 159) dimension of the Span of these vectors = Rank of the matrix

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \xrightarrow{-R_1+R_3} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

Swap R_2 and 3

$$\longrightarrow \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} \xrightarrow{R_2+R_3} \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

Rank = 3. D

#8, Page 15) Find a basis for the null space of A.

$$\left[\begin{array}{ccc|c} 2 & -1 & -2 & 0 \\ -4 & 2 & 4 & 0 \\ -8 & 4 & 8 & 0 \end{array}\right] \xrightarrow[4R_1+R_3]{2R_1+R_2}$$

$$\left[\begin{array}{ccc|c} 2 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad 2x_1 - x_2 - 2x_3 = 0$$

$$x_2 = 2x_1 - 2x_3 \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ 2x_1 - 2x_3 \\ x_3 \end{bmatrix}$$

$$= x_1 \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

So $v_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ would be a basis

but ~~at~~ they are not among the answers.

However $v_1 + v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ So

B

$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ is also a basis. which is the only one with two vectors.

#9, Page 161 A is $m \times n$. The system

$Ax = b$ has a solution for any

b if and only if the reduced row

echelon form of A does not have

a row of zeros. This means

that the rank of A is maximum

Rank $A = m = \#$ of rows of A.

E

#7, Page 170 $y^{(4)} - 8y'' + 16y = 0$ $y = e^{rx}$

$r^4 - 8r^2 + 16 = 0$ $(r^2 - 4)^2 = 0$

$(r-2)^2(r+2)^2 = 0$ roots $r = 2$ multiplicity 2

$r = -2$ multip 2.

General Solution

$$\boxed{y(x) = c_1 e^{2x} + c_2 x e^{2x} + c_3 e^{-2x} + c_4 x e^{-2x}.}$$

C

#8, Page 17[redacted])

$y'' + 9y' + 18y = 0 \qquad y = e^{rx}$

$r^2 + 9r + 18 = 0 \qquad (r+6)(r+3) = 0 \qquad r = -3, \; r = -6$

$y(x) = c_1 e^{-3x} + c_2 e^{-6x} \qquad y(0) = c_1 + c_2 = 0$

$y'(x) = -3c_1 e^{-3x} - 6c_2 e^{-6x} \qquad y'(0) = -3c_1 - 6c_2 = 3$

$c_1 = -c_2 = 1 \qquad \boxed{y(x) = e^{-3x} - e^{-6x}}$

$y(\tfrac{1}{3}\ln 3) = e^{-\ln 3} - e^{-2\ln 3} = \frac{1}{3} - \frac{1}{9} = \frac{2}{9}$ (A)

#16, Page 18) Find the rank of $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ -1 & -2 & -3 & -1 \end{bmatrix}$

$\xrightarrow[R_1+R_3]{-2R_1+R_2} \begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ Rank = 1.

dim of null space = # Columns − Rank

$= 4 - 1 = 3.$ (B)

#3, Page 20) The rank of A = # of non-zero rows of B = 3

dimension of Null (A) = # Columns − Rank = 5 − 3 = 2.

I is false.

II) is also false.

III) Is true.

IV) The dimension of the Column space = Rank = 3 so it cannot be $\mathbb{R}^4$.

E

#4, Page 21) $X = (x_1, x_2, x_3, x_4)$ denote vectors in $\mathbb{R}^4$

If $x_1 = x_3$

$$X = (x_1, x_2, x_1, x_4) = x_1(1,0,1,0) + x_2(0,1,0,0) + x_4(0,0,0,1)$$

Dimension is 3.

C

#6, Page 21)

Rank A = Dim of Row(A) = Dim Col(A).

I is true.

Dim of null A = # of Columns − Rank.

Dim of null A + Rank = # of Columns of A.

II is false in general. It is only true if A is a square matrix.

III) Rank of A = Dim of Col (A) = # of columns which contains a pivot.

(1st non-zero term of a non-zero row in an echelon form of A).

Dim of Null Space = # of Columns − # of Columns which contain a pivot

= # of columns which do not contain a pivot

III is true.

C

#3, Page 23) A is a 7×10 matrix.

Dim of null $(A) = 10 - \text{Rank}(A) = x$

$\text{Rank}(A) = 5$, $\boxed{x = 5}$.

A^T is a 10×7 matrix $\text{Rank}(A^T) = \text{Rank}(A)$

Dim of null $(A^T) = 7 - \text{Rank}(A^T) = 7 - 5 = 2$

$x = 5, \quad y = 2$ E

#1, Page 24) The vector is in the Span of the other two if there are non-zero constants c_1, c_2 and c_3 such that $c_1 V_1 + c_2 V_2 + c_3 V_3 = 0$

In other words the system has a non zero solution.

$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 0 \\ 3 & 2 & 2 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & -1 & k & 0 \end{array}\right] \xrightarrow[R_1 + R_4]{\substack{2R_1 + R_2 \\ R_1 + R_3}}$$

$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 0 \\ 0 & 4 & 6 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & 0 & k+2 & 0 \end{array}\right] \xrightarrow{-2R_3 + R_2} \left[\begin{array}{ccc|c} -1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & 0 & k+2 & 0 \end{array}\right]$$

Continuation #1, Page 24) ~~Swap~~ Move R2 to the bottom.

$$\left[\begin{array}{ccc|c} -1 & 1 & 2 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & 0 & k+2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \text{If } k+2 \neq 0$$

the only solution is $c_1 = c_2 = c_3 = 0$

So the vector is a linear combination of the other two if $k = -2$.

#11, Page 25) $y'' + 2y' + 2y = 0 \qquad y = e^{rx}$

$$r^2 + 2r + 2 = 0 \qquad r = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$r = \frac{-2 \pm \sqrt{-4}}{2} \qquad r = -1 \pm i$$

So we have two solution: $y_1(x) = e^{-x} \cos x$

$$y_2(x) = e^{-x} \sin x.$$

$$y(x) = c_1 e^{-x} \cos x + c_2 e^{-x} \sin x.$$

$y(0) = c_1 = 1; \qquad y'(0) = -c_1 + c_2 = 0 \qquad c_2 = 1$

$$\boxed{y(x) = e^{-x} \cos x + e^{-x} \sin x}$$

E

12) Page 25

$$y'' - 6y' + 9y = 0$$

$y = e^{rx}$ $\quad r^2 - 6r + 9 = 0 \quad (r-3)^2 = 0$

Solutions $\quad y_1(x) = e^{3t}, \quad y_2(x) = t e^{3t}$

General Solution $\quad y(x) = e^{3t}(C_1 + C_2 t).$

~~Solutio~~ Equation has two L. I solution.

$y_1 = e^{3t}$ and $y_2 = t e^{3t}$

9) Page 26.

$$y'' + 2y' + y = 0$$

$$y(0) = 3, \quad y'(0) = -2$$

$y = e^{rt}$ $\quad r^2 + 2r + 1 = 0 = (r+1)^2 \quad r = -1$

$$y(x) = e^{-x}(C_1 + C_2 x)$$

$$y'(x) = -e^{-x}(C_1 + C_2 x) + C_2 e^{-x}$$

$y(0) = C_1 = 3;$

$y'(0) = C_2 - C_1 = -2 \quad C_2 = 1.$

$y(x) = e^{-x}(3 + x)$ $\quad \boxed{y(1) = 4e^{-1}}$ $\quad$ D

11 Page 27) The homogeneous equation is.

$$y'' - 4y' + 4y = 0 \qquad y = e^{rx}$$

$$r^2 - 4r + 4 = (r-2)^2 \quad \text{so } r = 2 \text{ has mult. } 2$$

$$y_1(x) = e^{2x}; \quad y_2 = xe^{2x}.$$

The right hand side of the equation is

$$F(x) = (x+1)e^{2x} + x^2 e^x + \cos x.$$

The term e^x is not a solution of the homogeneous eq. So $x^2 e^x$ contributes with a term

$$(b_1 x^2 + b_2 x + b_3)e^x \quad \text{to } y_p$$

$\cos x$ is not a solution of the homogeneous eq. So it contributes.

$$C_1 \cos x + C_2 \sin x.$$

e^{2x} is a solution of the homogeneous eq and $r=2$ has multiplicity 2. So $(x+1)e^{2x}$ contributes with a term

$$x^2(a_1 x + a_2)$$

11, Page 27) Continuation: So

$$y_p(x) = x^2\,(a_1 x + a_2) + (b_1 x^2 + b_2 x + b_3)\, e^x + C_1 \cos x + C_2 \sin x$$

C

7, Page 28. Basis for $Ax = 0$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_1 + x_2 + x_3 = 0 \qquad x_1 = -x_2 - x_3$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -x_2 - x_3 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Basis $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

A

10, Page 30)

$$y^{(5)} + 4y^{4} + 5y^{(3)} = 0$$

$y = e^{rx}$ $\qquad r^5 + 4r^4 + 5r^3 = r^3(r^2 + 4r + 5) = 0$

$r = 0$ has multiplicity 3 , $y_1(x) = 1$, $y_2(x) = x$, $y_3 = x^2$

$r^2 + 4r + 5 = 0$ $\qquad r = -\frac{4 \pm \sqrt{16 - 20}}{2} = \frac{-2 \pm 2i}{2}$

$r = -1 + i$, $r = -1 - i$

So the solutions are $y_4 = e^{-x} \cos x$, $y_5 = e^{-x} \sin x$

$$y_h(x) = C_1 + C_2 x + C_3 x^2 + C_4 e^{-x} \cos x + C_5 e^{-x} \sin x$$

B

10. Find conditions for a, b and c such that the vector $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ is in the column space of the matrix

$$\begin{bmatrix} 3 & 1 & 2 \\ 2 & 5 & 8 \\ 1 & -4 & -6 \end{bmatrix}$$

A. $a+b+c=0$

B. $a-b-c=0$

C. $2a-b+c=0$

D. $3a+b-2c=0$

E. $8a+b+c=0$

Solution: $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ is in the column space of this matrix if there exist c_1, c_2 and c_3 such that

$$c_1\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ 5 \\ -4 \end{bmatrix} + c_3\begin{bmatrix} 2 \\ 8 \\ -6 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

or if the system has a solution. So, we form the augmented matrix

$$\begin{bmatrix} 3 & 1 & 2 & a \\ 2 & 5 & 8 & b \\ 1 & -4 & -6 & c \end{bmatrix} \xrightarrow[\text{1 and 3}]{\text{Switch rows}} \begin{bmatrix} 1 & -4 & -6 & c \\ 2 & 5 & 8 & b \\ 3 & 1 & 2 & a \end{bmatrix}$$

$$\xrightarrow[-3R_1+R_3]{-2R_1+R_2} \begin{bmatrix} 1 & -4 & -6 & c \\ 0 & 13 & 20 & b-2c \\ 0 & 13 & 20 & a-3c \end{bmatrix} \xrightarrow{-R_2+R_3}$$

$$\begin{bmatrix} 1 & -4 & -6 & c \\ 0 & 13 & 20 & b-2c \\ 0 & 0 & 0 & a-b-c \end{bmatrix}$$

the system has a solution if and only if $\boxed{a-b-c=0}$

B